5–71. The rod assembly is used to support the 250-lb cylinder. Determine the components of reaction at the ball-and-socket joint \( A \), the smooth journal bearing \( E \), and the force developed along rod \( CD \). The connections at \( C \) and \( D \) are ball-and-socket joints.

**Equations of Equilibrium:** Since rod \( CD \) is a two-force member, it exerts a force \( F_{DC} \) directed along its axis as defined by \( a_{DC} \) on rod \( BC \), Fig. a. Expressing each of the forces indicated on the free-body diagram in Cartesian vector form,

\[
\begin{align*}
F_A &= A_x i + A_y j + A_z k \\
F_E &= E_x i + E_y j + E_z k \\
W &= (-250k) \text{ lb} \\
F_{DC} &= -F_{DC} k
\end{align*}
\]

Applying the force equation of equilibrium

\[
\sum \mathbf{F} = 0; \quad F_A + F_E + F_{DC} + W = 0
\]

\[
(A_x i + A_y j + A_z k) + (E_x i + E_y j + E_z k) + (-F_{DC} k) + (-250k) = 0
\]

\[
(A_x + E_x) i + (A_y + E_y) j + A_z - F_{DC} k = 0
\]

Equating \( i \), \( j \), and \( k \) components,

\[
\begin{align*}
A_x + E_x &= 0 \quad & (1) \\
A_y &= 0 \quad & (2) \\
A_z - F_{DC} - 250 &= 0 \quad & (3)
\end{align*}
\]

In order to apply the moment equation of equilibrium about point \( A \), the position vectors \( r_{AC}, r_{AE}, \) and \( r_{AF} \), Fig. a, must be determined first.

\[
r_{AC} = [-1i + 1j] \text{ ft}
\]

\[
r_{AE} = [2i] \text{ ft}
\]

\[
r_{AF} = [1.5i + 3j] \text{ ft}
\]

Thus,

\[
\sum \mathbf{M}_A = 0; \quad (r_{AC} \times F_{DC}) + (r_{AE} \times F_E) + (r_{AF} \times W) = 0
\]

\[
(-1i + 1j) \times (-F_{DC} k) + (2i) \times (E_x i + E_y j + E_z k) + (1.5i + 3j) \times (-250k) = 0
\]

\[
(-F_{DC} + 2E_y - 750k) j + (375 - F_{DC}) i + (-2E_z) k = 0
\]
Equating i, j, and k components,

\[-F_{PC} + 2E_z = 750 = 0 \quad (4)\]
\[375 - F_{PC} = 0 \quad (5)\]
\[-2E_z = 0 \quad (6)\]

Solving Eq. (1) through (6), yields

\[F_{PC} = 375 \text{ lb} \quad \text{Ans.}\]
\[E_x = 0 \quad \text{Ans.}\]
\[E_z = 362.5 \text{ lb} \quad \text{Ans.}\]
\[A_x = 0 \quad \text{Ans.}\]
\[A_y = 0 \quad \text{Ans.}\]
\[A_z = 62.5 \text{ lb} \quad \text{Ans.}\]
5–72. Determine the components of reaction acting at the smooth journal bearings $A$, $B$, and $C$.

**Equations of Equilibrium:** From the free-body diagram of the shaft, Fig. 5. $C_y$ and $C_z$ can be obtained by writing the force equation of equilibrium along the $y$ axis and the moment equation of equilibrium about the $y$ axis.

\[
\Sigma F_y = 0; \quad 450 \cos 45^\circ + C_y = 0
\]
\[
C_y = -318.20 \text{ N} = -318 \text{ N} \quad \text{ Ans.}
\]
\[
\Sigma M_y = 0; \quad C_z(0.6) - 300 = 0
\]
\[
C_z = 500 \text{ N} \quad \text{ Ans.}
\]

Using the above results and writing the moment equations of equilibrium about the $x$ and $z$ axes,

\[
\Sigma M_x = 0; \quad B_z(0.8) - 450 \cos 45^\circ(0.4) = 0
\]
\[
B_z = -272.70 \text{ N} = -273 \text{ N} \quad \text{ Ans.}
\]
\[
\Sigma M_z = 0; \quad -B_z(0.8) - (-318.20)(0.6) = 0
\]
\[
B_z = 238.65 \text{ N} = 239 \text{ N} \quad \text{ Ans.}
\]

Finally, using the above results and writing the force equations of equilibrium along the $x$ and $y$ axes,

\[
\Sigma F_x = 0; \quad A_x + 238.5 = 0
\]
\[
A_x = -238.65 \text{ N} = -239 \text{ N} \quad \text{ Ans.}
\]
\[
\Sigma F_y = 0; \quad A_y - (-272.70) + 500 - 450 \sin 45^\circ = 0
\]
\[
A_y = 90.90 \text{ N} = 90.9 \text{ N} \quad \text{ Ans.}
\]

The negative signs indicate that $C_y$, $B_z$, and $A_x$ act in the opposite sense of that shown on the free-body diagram.
5-74. If the load has a weight of 200 lb, determine the \( x, y, z \) components of reaction at the ball-and-socket joint \( A \) and the tension in each of the wires.

Equations of Equilibrium: Expressing the forces indicated on the free-body diagram, Fig. a, in Cartesian vector form,

\[
\begin{align*}
\mathbf{F}_A &= A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} \\
W &= [-200] \text{ lb} \\
\mathbf{F}_{BD} &= F_{BD} \mathbf{k} \\
\mathbf{F}_{CD} &= F_{CD} \mathbf{k} \\
\mathbf{F}_{EF} &= F_{EF} \mathbf{k}
\end{align*}
\]

Applying the force equation of equilibrium,

\[
\Sigma F = 0 \quad \Rightarrow \quad \mathbf{F}_A + \mathbf{F}_{BD} + \mathbf{F}_{CD} + \mathbf{F}_{EF} + \mathbf{W} = 0
\]

\[
\begin{align*}
(A_x i + A_y j + A_z k) + F_{BD} k + \left(- \frac{4}{5} F_{CD} j + \frac{3}{5} F_{CD} k\right) + F_{EF} k + (-200) k &= 0 \\
A_x i + A_y j + A_z k &= 0
\end{align*}
\]

Equating \( i, j, \) and \( k \) components,

\[
\begin{align*}
A_x &= 0 \\
A_y - \frac{4}{5} F_{CD} &= 0 \\
A_z + F_{BD} + \frac{3}{5} F_{CD} + F_{EF} - 200 &= 0
\end{align*}
\]

In order to write the moment equation of equilibrium about point \( A \), the position vectors \( \mathbf{r}_{AB}, \mathbf{r}_{AG}, \mathbf{r}_{AC}, \) and \( \mathbf{r}_{AG} \) must be determined first.

\[
\begin{align*}
\mathbf{r}_{AB} &= [41] \text{ ft} \\
\mathbf{r}_{AG} &= [41+2] \text{ ft} \\
\mathbf{r}_{AC} &= [41+4] \text{ ft} \\
\mathbf{r}_{AG} &= [21+4] \text{ ft}
\end{align*}
\]

Thus,

\[
\begin{align*}
\Sigma M_A &= \theta \left( (\mathbf{r}_{AB} \times \mathbf{F}_{BD}) + (\mathbf{r}_{AC} \times \mathbf{F}_{CD}) + (\mathbf{r}_{AE} \times \mathbf{F}_{EF}) + (\mathbf{r}_{AG} \times \mathbf{W})\right) = 0 \\
&= (41) \times (F_{BD} k) + (41+4) \times \left(- \frac{4}{5} F_{CD} j + \frac{3}{5} F_{CD} k\right) + (21+4) \times (F_{EF} k) + (41+4) \times (-200 k) \\
&= \left(\frac{12}{5} F_{CD} + 4 F_{EF} - 400\right) j + \left(-8 F_{BD} - \frac{12}{5} F_{CD} - 2 F_{EF} + 800\right) k = 0
\end{align*}
\]
Equating \( i, j, \) and \( k \) components,

\[
\frac{12}{5} F_{CD} + 4F_{EF} - 400 = 0
\]  \hspace{1cm} (4)

\[
-4F_{BD} - \frac{12}{15} F_{CD} - 2F_{EF} + 800 = 0
\]  \hspace{1cm} (5)

\[
-\frac{16}{5} F_{CD} = 0
\]  \hspace{1cm} (6)

Solving Eqs. (1) through (6),

\[ F_{CD} = 0 \]
\[ F_{EF} = 100 \text{ lb} \]
\[ F_{BD} = 150 \text{ lb} \]
\[ A_x = 0 \]
\[ A_y = 0 \]
\[ A_z = 100 \text{ lb} \]

The negative signs indicate that \( A_x \) acts in the opposite sense to that on the free-body diagram.
5–78. The plate has a weight of \( W \) with center of gravity at \( G \). Determine the tension developed in wires \( AB \), \( CD \), and \( EF \) if the force \( P = 0.75W \) is applied at \( d = L/2 \).

**Equations of Equilibrium:** From the free-body diagram, Fig. 44918, \( T_{AB} \) can be obtained by writing the moment equation of equilibrium about the \( x' \) axis.

\[
\sum M_{x'} = 0; \quad 0.75W \left( \frac{L}{2} + \frac{L}{2} \cos 45^\circ \right) + W \left( \frac{L}{2} \right) - T_{AB} (L) = 0
\]

\[
T_{AB} = 1.1402W = 1.14W \quad \text{Ans.}
\]

Using the above result and writing the moment equations of equilibrium about the \( y \) and \( y' \) axes,

\[
\sum M_y = 0; \quad W \left( \frac{L}{2} \right) + 0.75W \left( \frac{L}{2} + \frac{L}{2} \sin 45^\circ \right) - 1.1402W \left( \frac{L}{2} \right) - T_{EF} (L) = 0
\]

\[
T_{EF} = 0.570W \quad \text{Ans.}
\]

\[
\sum M_{y'} = 0; \quad T_{CD} (L) + 1.1402W \left( \frac{L}{2} \right) - W \left( \frac{L}{2} \right) - 0.75W \left( \frac{L}{2} - \frac{L}{2} \sin 45^\circ \right) = 0
\]

\[
T_{CD} = 0.0396W \quad \text{Ans.}
\]
7–18. Determine the internal normal force, shear force, and moment at points D and E in the overhang beam. Point D is located just to the left of the roller support at B, where the couple moment acts.

The intensity of the triangular distributed load at E can be found using the similar triangles in Fig. b.

With reference to Fig. a,

\[ + \Sigma M_A = 0; \quad B_y(3) - 2(3)(1.5) - 6 - \frac{1}{2}(2)(3)(4) - \left( \frac{2}{3} \right)(6) = 0 \]

\[ B_y = 15 \text{kN} \]

Using this result and referring to Fig. c,

\[ \sum F_x = 0; \quad 5 \left( \frac{4}{3} \right) - N_D = 0 \quad N_D = 4 \text{kN} \quad \text{Ans.} \]

\[ + \Sigma F_y = 0; \quad V_D + 15 - \frac{1}{2}(2)(3) - \left( \frac{3}{5} \right)(6) = 0 \quad V_D = -9 \text{kN} \quad \text{Ans.} \]

\[ \Sigma M_D = 0; \quad -M_D - \frac{1}{2}(2)(3)(1) - \left( \frac{3}{5} \right)(3) = 0 \quad M_D = -18 \text{kN} \cdot \text{m} \quad \text{Ans.} \]

Also, by referring to Fig. d, we can write

\[ \sum F_x = 0; \quad 5 \left( \frac{4}{3} \right) - N_E = 0 \quad N_E = 4 \text{kN} \quad \text{Ans.} \]

\[ + \Sigma F_y = 0; \quad V_E - \frac{1}{2}(1)(1.5) - \left( \frac{3}{5} \right)(1.5) = 0 \quad V_E = 3.75 \text{kN} \quad \text{Ans.} \]

\[ \Sigma M_E = 0; \quad -M_E - \frac{1}{2}(1)(1.5)(0.5) - \left( \frac{3}{5} \right)(1.5) = 0 \quad M_E = -4.875 \text{kN} \cdot \text{m} \quad \text{Ans.} \]

The negative sign indicates that \( V_D, M_D, \) and \( M_E \) act in the opposite sense to that shown on the free-body diagram.
7–37. The shaft is supported by a thrust bearing at A and a journal bearing at B. Determine the x, y, z components of internal loading at point C.

With reference to Fig. a,

\[ \Sigma M_x = 0; \quad B_x (3 - 900(1)) - 750(1) = 0 \quad B_x = 550 \text{ N} \]
\[ \Sigma M_z = 0; \quad 750(2) + 600(2) - B_z (3) = 0 \quad B_z = 900 \text{ N} \]

Using these results and referring to Fig. b,

\[ \Sigma F_x = 0; \quad (V_C)_x + 900 - 750 - 600 = 0 \quad (V_C)_x = 450 \text{ N} \quad \text{Ans.} \]
\[ \Sigma F_y = 0; \quad N_C = 0 \quad \text{Ans.} \]
\[ \Sigma F_z = 0; \quad (V_C)_z + 550 = 0 \quad (V_C)_z = -550 \text{ N} \quad \text{Ans.} \]

\[ \Sigma M_x = 0; \quad (M_C)_x + 550(1.5) = 0 \quad (M_C)_x = -825 \text{ N} \cdot \text{m} \quad \text{Ans.} \]
\[ \Sigma M_y = 0; \quad T_C + 600(0.2) - 750(0.2) = 0 \quad T_C = 30 \text{ N} \cdot \text{m} \quad \text{Ans.} \]
\[ \Sigma M_z = 0; \quad (M_C)_z + 750(0.5) + 600(0.5) - 900(1.5) = 0 \quad (M_C)_z = 675 \text{ N} \cdot \text{m} \quad \text{Ans.} \]

The negative signs indicate that \((V_C)_z\) and \((M_C)_z\) act in the opposite sense to those shown in the free-body diagram.
•7–57. Draw the shear and moment diagrams for the overhang beam.

Since the loading is discontinuous at support $B$, the shear and moment equations must be written for regions $0 \leq x < 3$ m and $3$ m $< x \leq 6$ m of the beam. The free-body diagram of the beam’s segment sectioned through an arbitrary point within these two regions is shown in Figs. b and c.

Region $0 \leq x < 3$ m, Fig. b

\[ + \Sigma F_y = 0; \quad -4 - \frac{1}{2} \left( \frac{4}{3} x \right) - V = 0 \]
\[ V = \left\{ -\frac{2}{3} x^2 - 4 \right\} \text{kN} \quad (1) \]

\[ + \Sigma M = 0; \quad M + \frac{1}{2} \left( \frac{4}{3} x \right) \left( \frac{2}{3} x \right) + 4x = 0 \quad M = \left\{ -\frac{2}{9} x^3 - 4x \right\} \text{kN} \cdot \text{m} \quad (2) \]

Region $3$ m $< x \leq 6$ m, Fig. c

\[ + \Sigma F_y = 0; \quad V - 4(6-x) = 0 \]
\[ V = \{24 - 4x\} \text{kN} \quad (3) \]

\[ + \Sigma M = 0; \quad -M - 4(6-x) \left[ \frac{1}{2} (6-x) \right] = 0 \quad M = \{-2(6-x)^2\} \text{kN} \cdot \text{m} \quad (4) \]

The shear diagram shown in Fig. d is plotted using Eqs. (1) and (3). The value of shear just to the left and just to the right of the support is evaluated using Eqs. (1) and (3), respectively.

\[ V_{l x=3 \text{ m}} = -\frac{2}{3} (3^2) - 4 = -10 \text{ kN} \]
\[ V_{r x=3 \text{ m}} = 24 - 4(3) = 12 \text{ kN} \]

The moment diagram shown in Fig. e is plotted using Eqs. (2) and (4). The value of the moment at support $B$ is evaluated using either Eq. (2) or Eq. (4).

\[ M_{l x=3 \text{ m}} = -\frac{2}{9} (3^2) - 4(3) = -18 \text{ kN} \cdot \text{m} \]

or

\[ M_{l x=3 \text{ m}} = -2(6-3)^2 = -18 \text{ kN} \cdot \text{m} \]
7–59. Determine the largest intensity $w_0$ of the distributed load that the beam can support if the beam can withstand a maximum bending moment of $M_{\text{max}} = 20 \text{kN} \cdot \text{m}$ and a maximum shear force of $V_{\text{max}} = 80 \text{kN}$.

Since the loading is discontinuous at support $B$, the shear and moment equations must be written for regions $0 \leq x < 4.5 \text{ m}$ and $4.5 \text{ m} < x \leq 6 \text{ m}$ of the beam. The free-body diagram of the beam’s segment sectioned through the arbitrary points within these two regions are shown in Figs. b and c.

Region $0 \leq x < 4.5 \text{ m}$, Fig. b

\[ + \sum F_y = 0; \quad 2.167w_0 - w_0x - V = 0 \quad V = w_0(2.167 - x) \quad (1) \]

\[ + \sum M = 0; \quad M + w_0x\left(\frac{x}{2}\right) - 2.167w_0x = 0 \quad M = w_0(2.167x - 0.5x^2) \quad (2) \]

Region $4.5 \text{ m} < x \leq 6 \text{ m}$, Fig. c

\[ + \sum F_y = 0; \quad V - \frac{1}{3}\left(\frac{6-x}{1.5}\right)w_0(6-x) = 0 \quad V = \frac{w_0}{3}(6-x)^2 \quad (3) \]

\[ + \sum M = 0; \quad -M - \frac{1}{3}\left(\frac{6-x}{1.5}\right)w_0(6-x)\left[\frac{1}{3}(6-x)\right] = 0 \quad M = -\frac{w_0}{9}(6-x)^3 \quad (4) \]

The shear diagram shown in Fig. d is plotted using Eqs. (1) and (3). The value of the shear just to the left and right of support $B$ is evaluated using either Eq. (1) or Eq. (3), respectively.

\[ V_{x=4.5 \text{ m}^-} = w_0(2.167 - 4.5) = -2.333w_0 \]

\[ V_{x=4.5 \text{ m}^+} = \frac{w_0}{3}(6 - 4.5)^2 = 0.75w_0 \]

The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

\[ 0 = w_0(2.167 - x) \quad x = 2.167 \text{ m} \]

The moment diagram shown in Fig. e is plotted using Eqs. (2) and (4). The value of the moment at $x = 2.167 \text{ m}$ ($V = 0$) is evaluated using Eq. (2).

\[ M_{x=2.167 \text{ m}} = w_0\left[2.167(2.167) - 0.5(2.167)^2\right] = 2.347w_0 \]

The value of the moment at support $B$ is evaluated using Eqs. (2) or (4).

\[ M_{x=4.5 \text{ m}} = -\frac{w_0}{9}(6 - 4.5)^3 = -0.375w_0 \]

By observing the shear and moment diagrams, we notice that $V_{\text{max}} = 2.333w_0$ and $M_{\text{max}} = 2.347w_0$. Thus,

$V_{\text{max}} = 80 = 2.333w_0$

$w_0 = 34.29 \text{kN} / \text{m}$

$M_{\text{max}} = 20 = 2.347w_0$

$w_0 = 8.52 \text{kN} / \text{m}$ (control!)

Ans.
\[ A_x = 0 \]
\[ A_y = 2.167w_0 \]
\[ B_y = 3.083w_0 \]

(a)

\[ 0 \leq x < 4.5 \text{ m} \] (b)

\[ \frac{1}{2}[(\frac{6-x}{1.5})w_0](6-x) \]
\[ \frac{1}{3}(6-x) \]

(c) \[ 4.5 \text{ m} \leq x \leq 6 \text{ m} \]

(d)

\[ 2.347w_0 \]

(e)